NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 168

THE GENERAL EFFICIENCY CURVE FOR AIR PROPELLERS

By WALTER S. DIEHL

WASHINGTON
GOVERNMENT PRINTING OFFICE
1923

13

BOLS.

IVED UNITS.

English.	
Unit.	Symbol.
foot (or mile) second (or hour) weight of one pound	ft. (or mi.). sec. (or hr.). lb.
horsepower mi/hr.	HP M. P. H.

, ETC.

weight of "standard" air, 1.223 kg/m.^{3} 07635 lb/ft.^{3}

t of inertia, mk^2 (indicate axis of the is of gyration, k, by proper subscript).

; wing area, S_w, etc.

; chord length, c.

ratio $= b/c$

ce from $c.\ g.$ to elevator hinge, f.

ent of viscosity, μ.

SYMBOLS.

al angle, γ

lds Number $= \rho \frac{Vl}{\mu}$, where l is a linear dision.

or a model airfoil 3 in. chord, 100 mi/hr., nal pressure, 0°C: 255,000 and at 15.6°C, 000;

a model of 10 cm. chord, 40 m/sec., esponding numbers are 299,000 and 000.

of pressure coefficient (ratio of distance P. from leading edge to chord length),

of stabilizer setting with reference to er wing. $(i_t - i_w) = \beta$

of attack, α

of downwash, ϵ

AERONAUTICAL SYM

1. FUNDAMENTAL AND DE

	Symbol.	Metric.	
		Unit.	Symbol.
Length Time Force	l t F	meter second weight of one kilogram	m. sec. kg.
Power Speed	P	kg.m/sec m/sec	 m. p. s.

2. GENERAL SYMBOL

Weight, $W = mg$.

Standard acceleration of gravity, $g = 9.806\text{m/sec.}^2 = 32.172\text{ft/sec.}^2$

Mass, $m = \frac{W}{g}$

Density (mass per unit volume), ρ

Standard density of dry air, 0.1247 (kg.-m.-sec.) at 15.6°C. and 760 mm. = 0.00237 (lb.-ft.-sec.)

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Area,

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3. AERODYNAMICAL

True airspeed, V

Dynamic (or impact) pressure, $q = \frac{1}{2} \rho V^2$

Lift, L; absolute coefficient $C_L = \frac{L}{qS}$

Drag, D; absolute coefficient $C_D = \frac{D}{qS}$.

Cross-wind force, C; absolute coefficient $C_C = \frac{C}{qS}$.

Resultant force, R

(Note that these coefficients are twice as large as the old coefficients L_c, D_c.)

Angle of setting of wings (relative to thrust line), i_w

Angle of stabilizer setting with reference to thrust line i_t

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Angle

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Angle

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By WALTER S. DIEHL
Bureau of Aeronautics, Navy Department

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THE GENERAL EFFICIENCY CURVE FOR AIR PROPELLERS.

By Walter S. Diehl.

SUMMARY.

This report, which was prepared for the National Advisory Committee for Aeronautics, is a study of propeller efficiency, based on the equation

$$\eta=\left(\frac{V}{\pi ND}\right)\cot(\varphi+\gamma)$$

where

V = speed of advance.
N = revolutions per unit of time.
D = diameter of the helix described by the particular element under consideration.

$$\varphi=\tan^{-1}\left(\frac{V}{\pi ND}\right)$$

and

$$\gamma=\tan^{-1}\left(\frac{D}{L}\right)$$

It is shown that this formula may be used to obtain a "general efficiency curve" in addition to the well-known maximum efficiency curve. These two curves, when modified somewhat by experimental data, enable performance calculations to be made without detailed knowledge of the propeller. The curves may also be used to estimate the improvement in efficiency due to reduction gearing, or to judge the performance of a new propeller design.

INTRODUCTION.

The efficiency of an element of a propeller blade is given by the well-known formula[1]:

$$\eta=\frac{V}{\pi ND}\cot(\varphi+\gamma) \qquad (1)$$

where

V = speed of advance.
N = revolutions per unit of time.
D = diameter of the helix described by the particular element under consideration.

$$\varphi=\tan^{-1}\left(\frac{V}{\pi ND}\right)$$

and

$$\gamma=\tan^{-1}\left(\frac{D}{L}\right).$$

An analysis of this formula shows that it not only may be used to predict the maximum efficiency obtainable under a given set of conditions; that is, at a specified $\frac{V}{ND}$, but that it also supplies a "general efficiency curve," applying to all propellers. The curves thus obtained,

[1] See B. A. C. A.; R. & M. No. 328, or any book on propeller design.

when modified somewhat by experimental data, determine the efficiency curve for the best propeller of the series which has maximum efficiency at any desired value of $\left(\frac{V}{ND}\right)$. Obviously these curves enable one to calculate performance of aircraft without further investigation into the properties of the propeller which is to be used, than to determine the $\left(\frac{V}{ND}\right)$ at which it is desired that the efficiency η have its maximum value.

In order to simplify the arithmetical work involved in the derivation of the general efficiency curves, the theoretical efficiency for the tip section, as given by (1), will be used for the theoretical average efficiency. The error involved in this substitution is usually of the order of 1 per cent, as shown by the comparative figures of Table I, which is compiled from a series given in "A Treatise on Airscrews" (Parks). It should be noted that the difference between the tip efficiency and the average efficiency is sensitive to changes in the plan form of the blades.

THEORETICAL MAXIMUM EFFICIENCY.

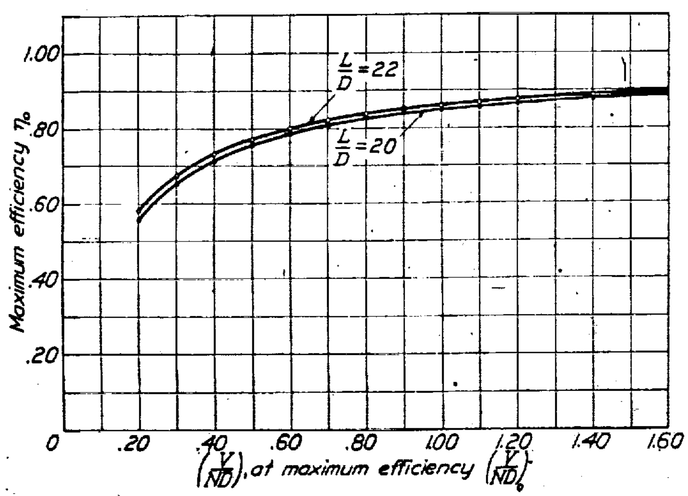

FIG. 1. Efficiency curves.

For all of the basic propeller blade sections in common use the maximum value of $\left(\frac{L}{D}\right)$ lies in the neighborhood of 20, say between 18 and 22. These limiting values correspond to $\gamma = 3° \, 09'$ and $\gamma = 2° \, 36'$, respectively. The value of φ is commonly greater than 5°. Consequently, for any given value of φ the probable variations in γ have only a small effect, so that the maximum efficiency is determined by φ and not by γ. Obviously the greater the value of φ the less important the variations in γ become.

Table II contains calculations for the values of theoretical maximum tip efficiencies corresponding to $\left(\frac{L}{D}\right)=20$ and $\left(\frac{L}{D}\right)=22$ for a wide range of $\left(\frac{V}{ND}\right)$. These efficiences are plotted against $\left(\frac{V}{ND}\right)$ in Fig. I, forming the familiar "efficiency curves."

PRACTICAL MAXIMUM EFFICIENCY.

In the preceding calculations for maximum efficiency, no allowance was made for indraft, interference, or variations in blade section and plan form. All of these factors affect the efficiency, and in some cases, adversely. The combined effect of their presence is more easily obtained from tests than from calculation. For this purpose, there is given in Table III the maximum efficiency and the $\left(\frac{V}{ND}\right)$ at which it occurs for each of the propellers tested by Durand and reported in N. A. C. A. Reports Nos. 14, 30, 64, and 109. These values are plotted as crosses in Figure 2, together with the theoretical curve for η vs. $\left(\frac{V}{ND}\right)$ when $\left(\frac{L}{D}\right)=22$.

It is immediately apparent from an inspection of Figure 2, that the maximum efficiencies obtained in test are consistently lower than the values which should theoretically be obtained for $\frac{L}{D}=22$. The difference decreases with $\left(\frac{V}{ND}\right)$ and the various test data points are so

grouped that a curve drawn through the maximum observed efficiency at each $\left(\frac{V}{ND}\right)$ will be quite similar to the theoretical curve. A curve so drawn, as on Figure 2, may be considered as the practical limit to maximum efficiency for propellers of conventional designs.

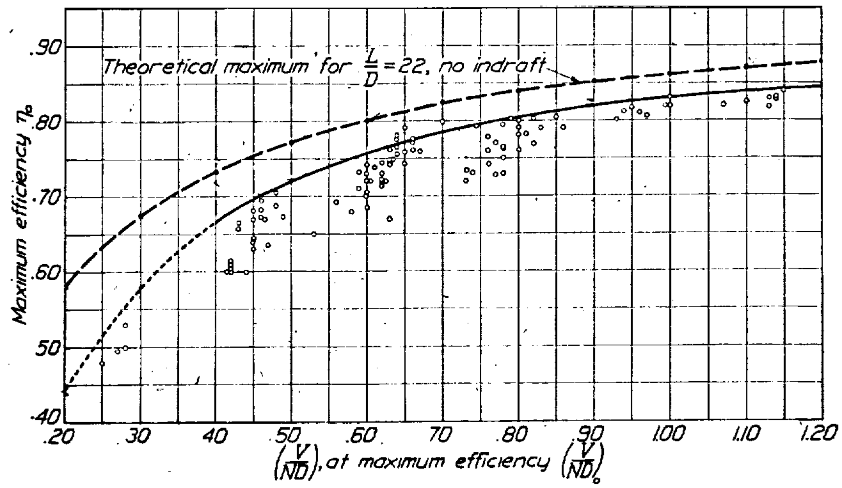

FIG. 2. Propeller efficiency. Variation of maximum efficiency with $\left(\frac{V}{ND}\right)$. From Durand's experiments.

THE GENERAL EFFICIENCY CURVE.

Denote by the subscript $_o$ the conditions corresponding to maximum efficiency, so that

$$\eta_o = \left(\frac{V}{\pi ND}\right)_o \times \cot\,(\varphi_o + \gamma_o) \tag{1a}$$

Then the ratio of the efficiency under any set of conditions to the maximum efficiency will be

$$\frac{\eta}{\eta_o} = \frac{\left(\frac{V}{\pi ND}\right)}{\left(\frac{V}{\pi ND}\right)_o} \frac{\cot\,(\varphi+\gamma)}{\cot\,(\varphi_o+\gamma_o)}$$

$$= \frac{\left(\frac{V}{\pi ND}\right)}{\left(\frac{V}{\pi ND}\right)_o} \frac{\tan\,(\varphi_o+\gamma_o)}{\tan\,(\varphi+\gamma)}$$

$$= \frac{\left(\frac{V}{\pi ND}\right)}{\left(\frac{V}{\pi ND}\right)_o} \left[\frac{\dfrac{\tan\varphi_o + \tan\gamma_o}{1 - \tan\varphi_o \tan\gamma_o}}{\dfrac{\tan\varphi + \tan\gamma}{1 - \tan\varphi\,\tan\gamma}}\right]$$

$$= \frac{\left(\frac{V}{\pi ND}\right)}{\left(\frac{V}{\pi ND}\right)_o} \left[\frac{\tan\varphi_o - \tan\varphi_o \tan\varphi \tan\gamma + \tan\gamma_o - \tan\gamma_o \tan\varphi \tan\gamma}{\tan\varphi - \tan\varphi \tan\varphi_o \tan\gamma_o + \tan\gamma - \tan\gamma \tan\varphi_o \tan\gamma_o}\right]$$

According to definition

$$\tan\gamma = \left(\frac{D}{L}\right)$$

$$\tan\gamma_o = \left(\frac{D}{L}\right)_o$$

$$\tan \varphi = \left(\frac{V}{\pi ND}\right)$$

$$\tan \varphi_0 = \left(\frac{V}{\pi ND}\right)_0$$

and substituting, one obtains

$$\frac{\eta}{\eta_0} = \frac{\left(\frac{V}{\pi ND}\right)}{\left(\frac{V}{\pi ND}\right)_0} \left[\frac{\left(\frac{D}{L}\right)_0 - \left(\frac{D}{L}\right)_0 \left(\frac{D}{L}\right) \left(\frac{V}{\pi ND}\right) + \left(\frac{V}{\pi ND}\right)_0 - \left(\frac{D}{L}\right) \left(\frac{V}{\pi ND}\right) \left(\frac{V}{\pi ND}\right)_0}{\left(\frac{D}{L}\right) - \left(\frac{D}{L}\right) \left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0 + \left(\frac{V}{\pi ND}\right) - \left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0 \left(\frac{V}{\pi ND}\right)} \right];$$

letting

$$\left(\frac{V}{\pi ND}\right) = R \left(\frac{V}{\pi ND}\right)_0$$

and grouping terms, one finds

$$\frac{\eta}{\eta_0} = R \left[\frac{1 - R\left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0}{1 - \left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0} \right] \cdot \left[\frac{\left(\frac{D}{L}\right)_0 + \left(\frac{V}{\pi ND}\right)_0}{\left(\frac{D}{L}\right) + R \left(\frac{V}{\pi ND}\right)_0} \right] \tag{2}$$

The value of $\left(\frac{D}{L}\right)_0$ is substantially constant for all tip sections in common use. For a representative section, No. 2 of the series given in Br ACA R&M #322, $\left(\frac{D}{L}\right)_0 = .0475$. The increase in $\left(\frac{D}{L}\right)$, or the decrease in $\left(\frac{L}{D}\right)$ is linear with angle of attack over a wide working range. For the section previously referred to $\left(\frac{D}{L}\right)$ varies from 0.0475 at 3° to 0.100 at 15° so that

$$\frac{\Delta \left(\frac{D}{L}\right)}{\Delta \alpha} = \frac{(0.100 - 0.0475)}{(15 - 3)} = .00437$$

Now, to a close approximation, the change in angle of attack is

$$\Delta \alpha = 57.3 \left[\left(\frac{V}{\pi ND}\right)_0 - \left(\frac{V}{\pi ND}\right) \right]$$

Therefore

$$\left(\frac{D}{L}\right) = \left(\frac{D}{L}\right)_0 + 0.25 \left[\left(\frac{V}{\pi ND}\right)_0 - \left(\frac{V}{\pi ND}\right) \right]$$

$$= \left(\frac{D}{L}\right)_0 + 0.25 \left(\frac{V}{\pi ND}\right)_0 [1 - R]$$

Substituting this in (2):

$$\frac{\eta}{\eta_0} = R \left[\frac{1 - R\left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0}{1 - \left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0} \right] \left[\frac{\left(\frac{D}{L}\right)_0 + \left(\frac{V}{\pi ND}\right)_0}{\left(\frac{D}{L}\right)_0 + \left(\frac{V}{\pi ND}\right)_0 (0.25 + 0.75\, R)} \right] \tag{2a}$$

Since $\left(\frac{D}{L}\right)_0 \left(\frac{V}{\pi ND}\right)_0$ will ordinarily be of the order of *.01*, the first term in brackets will be substantially unity and the equation may be written:

$$\frac{\eta}{\eta_0} = R \frac{\left(\frac{D}{L}\right)_0 + \left(\frac{V}{\pi ND}\right)_0}{\left(\frac{D}{L}\right)_0 + \left(\frac{V}{\pi ND}\right)_0 (0.25 + 0.75\, R)} \tag{2b}$$

From this equation alone it would be concluded that η/η_o for any value of R depended only on the value of $\left(\frac{V}{\pi ND}\right)_o$. For a particular value of R, say $R=0.5$, $\frac{\eta}{\eta_o}$ would vary from

$$\frac{\eta}{\eta_0}=R=0.5$$

when $\left(\frac{V}{\pi ND}\right)_o$ is very small, to

$$\frac{\eta}{\eta_o}=\frac{R}{(0.25+0.75\ R)}=0.8.$$

when $\left(\frac{V}{\pi ND}\right)_o$ is very large. Within the range of working values of $\left(\frac{V}{\pi ND}\right)_o$, which may be taken as 0.10 to 0.40 the variation in $\frac{\eta}{\eta_o}$ is between 0.67 and 0.75 (for $\left(\frac{D}{L}\right)_o=.0475$).

The preceding values do not take into consideration an important factor which has been purposely neglected up to this point. Referring to equation (1a), it will be noted that it was assumed that the maximum efficiency occurred when the value of $\left(\frac{V}{\pi ND}\right)$ was that which gave the tip section the angle of attack corresponding to the least value of $\left(\frac{D}{L}\right)_o$, $\left(\text{or the highest } \frac{L}{D}\right)$. It is almost superfluous to remark that near the maximum, the values of $\left(\frac{L}{D}\right)$, for any aerofoil, are substantially constant over a range of one or two degrees in angle of attack. Due to this characteristic, the maximum efficiency of a propeller designed for a low value of $\left(\frac{V}{\pi ND}\right)$ does not occur at the $\left(\frac{V}{\pi ND}\right)$ which gives the tip section, the angle of attack corresponding to its best $\left(\frac{L}{D}\right)$, but, since φ increases faster than cot $(\varphi+\gamma)$ decreases, the maximum efficiency will occur at a somewhat higher value of $\left(\frac{V}{\pi ND}\right)$. This effect may perhaps be made clearer by means of a numerical illustration. Take the case where $\left(\frac{D}{L}\right)_o=.0475$ and assume $\varphi=\gamma$. Then

$$\begin{aligned}\eta&=\left(\frac{V}{\pi ND}\right)\cdot\cot(\varphi+\gamma)\\&=.0475\cdot\cot(2^\circ\ 43'+2^\circ\ 43')\\&=.0475\times10.514\\&=.50\end{aligned}$$

and for a slightly greater value of $\left(\frac{V}{\pi ND}\right)$, say $\left(\frac{V}{\pi ND}\right)'=1.10\left(\frac{V}{\pi ND}\right)$ it will be found that $\left(\frac{D}{L}\right)$ has not changed appreciably, so that

$$\begin{aligned}\eta&=1.10\left(\frac{V}{\pi ND}\right)\cdot\cot(1.10\ \varphi+\gamma)\\&=.0522\cdot\cot(2^\circ\ 59'+2^\circ\ 43')\\&=.0522\times10.02\\&=.523.\end{aligned}$$

Now the effect of this characteristic is to remove almost entirely the differences in η/η_o noted previously; as the nominal value of $\left(\frac{V}{\pi ND}\right)_o$ is decreased, the actual value of $\left(\frac{V}{\pi ND}\right)_o$ (in terms of the nominal value) increases so that a higher value of η corresponds to a given value of R. For all practical purposes a single curve of $\frac{\eta}{\eta_o}$ vs $\left(\frac{V}{\pi ND}\right)/\left(\frac{V}{\pi ND}\right)_o$ applies to all

propellers, as may be seen by inspection of Tables IV and IV–A and Fig. 3. The tables contain calculations for two propellers rather widely separated in their characteristics, and the values of $\frac{\eta}{\eta_0}$ thus obtained lie on a single curve in Fig. 3. There is some divergence for values of R greater than 1.10 but this is ordinarily beyond the working range.

THE GENERAL EFFICIENCY CURVE GIVEN BY DURAND'S TESTS.

In Table V there are given values of η/η_0 vs $\left(\frac{V}{\pi ND}\right)/\left(\frac{V}{\pi ND}\right)_0$ for ten of Durand's propellers chosen at random but including the entire range of $\left(\frac{V}{\pi ND}\right)_0$ tested. The last column in this table gives the average for 45 propellers thus studied. This average does not differ appreciably from the average for 10 propellers.

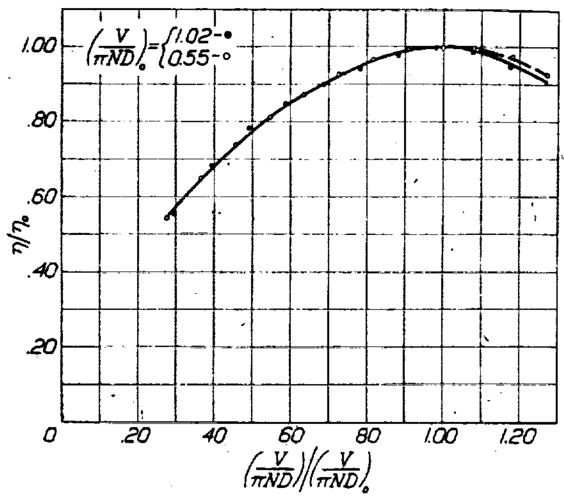

FIG. 3. Calculated curve η/η_0 vs. $\left(\frac{V}{\pi ND}\right)/\left(\frac{V}{\pi ND}\right)_0$. No allowance for indraft.

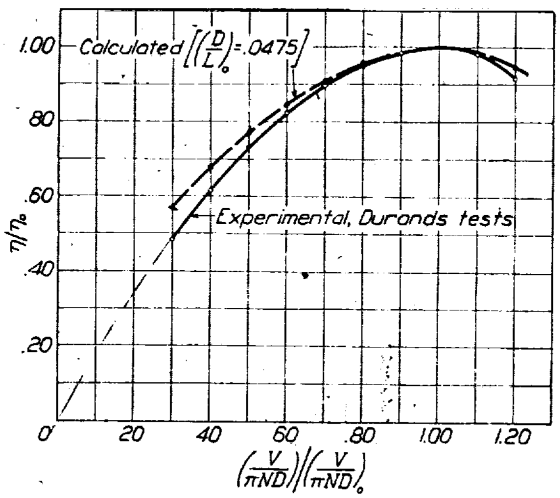

FIG. 4. Propeller efficiency. General curve.

It is to be noted that the deviations from the general average are surprisingly small, particularly over that part of the curve which could be used in normal flight. Part of the deviations are undoubtedly due to errors in reading values from the curves. In many cases it is difficult to determine the value of $\left(\frac{V}{ND}\right)_0$ accurately.

The experimental curve of η/η_0 vs. $\left(\frac{V}{\pi ND}\right)/\left(\frac{V}{\pi ND}\right)_0$ is plotted together with the calculated curve on Figure 4 for comparison. The differences are as expected both in magnitude and direction.

APPLICATIONS AND COMMENT.

It has been stated that, by the aid of the general efficiency curves, performance calculations may be made without detailed knowledge of the propeller which is to be used. The only data required is the value of $\left(\frac{V}{ND}\right)$ at which the maximum efficiency is desired to occur, and this is easily found. The value of the maximum efficiency is then determined by the solid curve on Figure 2, and the entire efficiency curve may be obtained, if required, by the use of the general efficiency curve of Figure 4.

To illustrate by a numerical example: assume $V=120$ *mi/hr.*, $N=1{,}800$ *r. p. m.*, and $D=8.0$ *ft.*, so that $\left(\frac{V}{ND}\right)_0=.735$. From Figure 2 the maximum efficiency corresponding to this

value of $\left(\frac{V}{ND}\right)_0$ is $\eta_0 = .793$. For the same propeller at $\left(\frac{V}{ND}\right) = .50$ $\left(\frac{V}{ND}\right) \Big/ \left(\frac{V}{ND}\right)_0 = .68$ and from Figure 4 the corresponding value of η/η_0 is 0.882. Therefore $\eta = .882 \times .793 = .70$. The efficiency at any other $\left(\frac{V}{ND}\right)$ is found in the same manner.

Further applications naturally suggest themselves. For example, the gain in efficiency due to the use of reduction gearing is readily obtained from Figure 2. The curves may also be used in the analysis of propeller characteristics to indicate the relative value of a particular design.

In using these curves it must be remembered that the solid curve on Figure 2 represents the *best* efficiency which, according to wind-tunnel tests, can be obtained at each value of $\left(\frac{V}{ND}\right)$. The actual maximum efficiency may be somewhat lower if the design be unfavorable, for example, in the case of a four-bladed propeller. The solid curve on Figure 4 is a general efficiency curve and applies to all propellers so far investigated, regardless of the value of the maximum efficiency or the value of $\left(\frac{V}{ND}\right)$ at which it occurs.

TABLE I.

Comparison of Average Efficiency and Tip Efficiency—Calculated Values.

WITHOUT INFLOW.			WITH INFLOW.		
$\frac{V}{ND}$	Tip efficiency.	Average efficiency.	$\frac{V}{ND}$	Tip efficiency.	Average efficiency.
0.20	0.45	0.43	0.20	0.26	0.261
.40	.67	.68	.40	.46	.457
.60	.79	.803	.60	.60	.610
.80	.815	.828	.80	.64	.662

Data taken from "A Treatise on Airscrews" (Park), pp. 55–63.

TABLE II.

Theoretical Maximum Efficiency.

$\frac{V}{ND}$	$\frac{V}{\pi ND}$	Φ	$\frac{L}{D}=20$			$\frac{L}{D}=22$		
			$(\Phi+\gamma)$	Cot $(\Phi+\gamma)$	η	$(\Phi+\gamma)$	Cot $(\Phi+\gamma)$	η
0.20	0.0637	3° 39′	6° 31′	8.754	0.557	6° 15′	9.131	0.582
.30	.0955	5° 27′	8° 19′	6.841	.653	8° 03′	7.071	.675
.40	.1273	7° 15′	10° 07′	5.605	.714	9° 51′	5.759	.734
.50	.1592	9° 03′	11° 55′	4.739	.754	11° 39′	4.850	.772
.60	.1910	10° 49′	13° 41′	4.107	.784	13° 25′	4.192	.800
.70	.2228	12° 34′	15° 26′	3.622	.807	15° 10′	3.689	.822
.80	.2546	14° 17′	17° 09′	3.241	.824	16° 53′	3.295	.838
.90	.2865	15° 59′	18° 51′	2.929	.839	18° 35′	2.974	.852
1.00	.3183	17° 39′	20° 31′	2.672	.850	20° 15′	2.711	.862
1.10	.3501	19° 18′	22° 10′	2.455	.859	21° 54′	2.488	.870
1.20	.3820	20° 54′	23° 46′	2.271	.867	23° 30′	2.300	.878
1.40	.4456	24° 01′	26° 53′	1.973	.879	26° 37′	1.996	.889
1.60	.5093	26° 59′	29° 51′	1.743	.888	29° 35′	1.762	.897
			$\gamma = \cot^{-1} 20 = 2°52′$			$\gamma = \cot^{-1} 22 = 2°36′$		

value of $\left(\frac{V}{ND}\right)$, is $\eta_o = .793$. For the same propeller at $\left(\frac{V}{ND}\right) = .50$ $\left(\frac{V}{ND}\right) / \left(\frac{V}{ND}\right)_o = .68$ and from Figure 4 the corresponding value of η/η_o is 0.882. Therefore $\eta = .882 \times .793 = .70$. The efficiency at any other $\left(\frac{V}{ND}\right)$ is found in the same manner.

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.30	.0955	5° 27′	8° 19′	6.841	.653	8° 03′	7.071	.675
.40	.1273	7° 15′	10° 07′	5.605	.714	9° 51′	5.759	.734
.50	.1592	9° 03′	11° 55′	4.739	.754	11° 39′	4.850	.772
.60	.1910	10° 49′	13° 41′	4.107	.784	13° 25′	4.192	.800
.70	.2228	12° 34′	15° 26′	3.622	.807	15° 10′	3.689	.822
.80	.2546	14° 17′	17° 09′	3.241	.824	16° 53′	3.295	.838
.90	.2865	15° 59′	18° 51′	2.929	.839	18° 35′	2.974	.852
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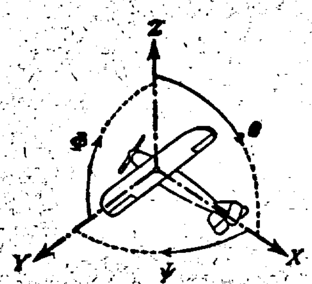

Positive directions of axes and angles (forces and moments) are shown by arrows.

Axis.		Force (parallel to axis) symbol.	Moment about axis.			Angle.		Velocities.	
Designation.	Symbol.		Designation.	Symbol.	Positive direction.	Designation.	Symbol.	Linear (component along axis).	Angular.
Longitudinal....	X	X	rolling.....	L	$Y \longrightarrow Z$	roll.....	Φ	u	p
Lateral.........	Y	Y	pitching...	M	$Z \longrightarrow X$	pitch....	Θ	v	q
Normal.........	Z	Z	yawing.....	N	$X \longrightarrow Y$	yaw.....	Ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{q\,b\,S} \quad C_m = \frac{M}{q\,c\,S} \quad C_n = \frac{N}{q\,f\,S}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS.

Diameter, D
Pitch (a) Aerodynamic pitch, p_a
(b) Effective pitch, p_e
(c) Mean geometric pitch, p_g
(d) Virtual pitch, p_v
(e) Standard pitch, p_s
Pitch ratio, p/D
Inflow velocity, V'
Slipstream velocity, V_s

Thrust, T
Torque, Q
Power, P
(If "coefficients" are introduced all units used must be consistent.)
Efficiency $\eta = T\,V/P$
Revolutions per sec., n; per min., N
Effective helix angle $\Phi = \tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS.

1 HP = 76.04 kg. m/sec. = 550 lb. ft/sec.
1 kg. m/sec. = 0.01315 HP
1 mi/hr. = 0.44704 m/sec.
1 m/sec. = 2.23693 mi/hr.

1 lb. = 0.45359 kg.
1 kg. = 2.20462 lb.
1 mi. = 1609.35 m. = 5280 ft.
1 m. = 3.28083 ft.